Equivalence of three topologies in spaces of Laplace-Stieltjes integrals

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Abstract

We study spaces generated by Laplace–Stieltjes integrals of the form

$$I(\sigma) = \int_1^{\infty} f(x)e^{x\sigma}\, dF(x), \quad \sigma \in \mathbb{R},$$

where F is a non-negative, nondecreasing, unbounded, right-continuous function on $(1, +\infty)$ and f is a real-valued function. Such integrals and their asymptotic behavior have been extensively studied in the literature (see, e.g., [1]).

We consider a class $U(F)$ of functions whose absolute values satisfy a regular variation condition with respect to F, and investigate the corresponding space $LS(U(F))$ of Laplace–Stieltjes integrals. Three natural topologies are introduced on $LS(U(F))$: a metric topology generated by a countable system of norms, and two additional topologies defined via paranorms associated with pointwise growth conditions (see [2]).

Under a mild condition on F ensuring sufficient decay of exponential integrals, we prove that these topologies are equivalent. This result provides a unified framework for studying convergence in $LS(U(F))$ and implies, in particular, uniform convergence on compact intervals.

Furthermore, we establish completeness of the space under these topologies and show that it forms a Fréchet space. Additional structural properties, including the Montel property, are also obtained, extending earlier results on functional spaces of Laplace–Stieltjes integrals [3].

References

[1] Sheremeta M. M., Asymptotical Behaviour of Laplace–Stieltjes Integrals, VNTL Publishers, Lviv, 2010.

[2] Maddox I. J., Elements of Functional Analysis, *Cambridge University Press*, 1988, 256 pp.

[3] Kuryliak A. O., Sheremeta M. M., On a Banach space and Fréchet spaces of Laplace–Stieltjes integrals, *Nonlinear Oscillations* **24** (2021), no. 2, 188–196.