# Degenerations of three-dimensional associative algebras

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Let $\mathbf{\Lambda}_3(\mathbb{C})\,(=\mathbb{C}^{27})$ be the space of structure vectors of 3-dimensional algebras over $\mathbb{C}$ considered as a $G$-module via the action of $G=\mathrm{GL}(3,\mathbb{C})$ on $\mathbf{\Lambda}_3(\mathbb{C})$ 'by change of basis'. We determine the complete degeneration picture inside the algebraic subset $\mathcal{A}_3^s$ of $\mathbf{\Lambda}_3(\mathbb{C})$ of associative algebra structures via the corresponding information on the algebraic subsets $\mathcal{L}_3$ and $\mathcal{J}_3$ of $\mathbf{\Lambda}_3(\mathbb{C})$ of Lie and Jordan algebra structures respectively. This is achieved with the help of certain $G$-module endomorphisms $\phi_1$, $\phi_2$ of $\mathbf{\Lambda}_3(\mathbb{C})$ which map $\mathcal{A}_3^s$ onto algebraic subsets of $\mathcal{L}_3$ and $\mathcal{J}_3$ respectively. (See [1].)

Moreover, we determine the complete degeneration picture inside the variety of nilpotent associative algebras of dimension 3 over an algebraically closed field, see [2]. Comparing with the discussion in [1], for some of the arguments in [2] alternative techniques needed to be developed which are now valid over an arbitrary algebraically closed field. There is a dichotomy of cases concerning the results obtained, corresponding to whether the characteristic of the field is 2 or not.

## References

[1] Ivanova N.M. and Pallikaros C.A., Degenerations of complex associative algebras of dimension three via Lie and Jordan algebras, *Advances in Group Theory and Applications*, **18** (2024), 41–79, arXiv:2212.10635.

[2] Ivanova N.M. and Pallikaros C.A., Degenerations of 3-dimensional nilpotent associative algebras over an algebraically closed field, *Ukr. Math. J.*, **76** (2025), 1808–1820, arXiv:2312.09771.